

PROBLEM 6-7a

Statement: The general linkage configuration and terminology for an offset fourbar slider-crank linkage are shown in Fig P6-2. The link lengths and the values of θ_2 and ω_2 are defined in Table P6-2. For row *a*, find the velocities of the pin joints *A* and *B* and the velocity of slip at the sliding joint using the analytic method. Draw the linkage to scale and label it before setting up the equations.

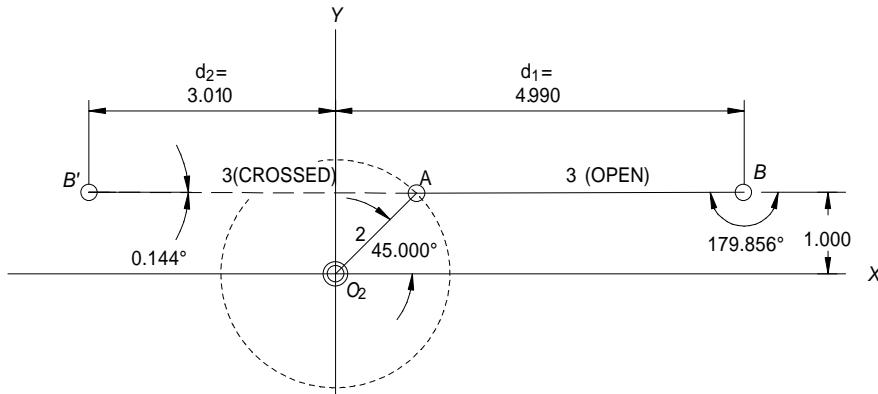
Given: Link lengths:

$$\text{Link 2 } (O_2 A) \quad a := 1.4 \quad \text{Link 3 } (AB) \quad b := 4 \quad \text{Offset } (y_B) \quad c := 1$$

$$\text{Crank angle} \quad \theta_2 := 45 \cdot \text{deg} \quad \text{Crank angular velocity} \quad \omega_2 := 10$$

Solution: See Figure P6-2 and Mathcad file P0607a.

1. Draw the linkage to scale and label it.



2. Determine θ_3 and d using equations 4.16 and 4.17.

$$\text{Crossed: } \theta_{32} := \arcsin\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right) \quad \theta_{32} = -0.144 \text{ deg}$$

$$d_2 := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_{32}) \quad d_2 = -3.010$$

$$\text{Open: } \theta_{31} := \arcsin\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi \quad \theta_{31} = 180.144 \text{ deg}$$

$$d_1 := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_{31}) \quad d_1 = 4.990$$

3. Determine the angular velocity of link 3 using equation 6.22a:

$$\text{Open} \quad \omega_{31} := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_{31})} \cdot \omega_2 \quad \omega_{31} = -2.475$$

$$\text{Crossed} \quad \omega_{32} := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_{32})} \cdot \omega_2 \quad \omega_{32} = 2.475$$

4. Determine the velocity of pin *A* using equation 6.23a:

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = -9.899 + 9.899i \quad |\mathbf{V}_A| = 14.000 \quad \arg(\mathbf{V}_A) = 135.000 \text{ deg}$$

5. Determine the velocity of pin B using equation 6.22b:

Open $V_{B1} = -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{31} \cdot \sin(\theta_{31})$ $V_{B1} = -9.875$

Crossed $V_{B2} = -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{32} \cdot \sin(\theta_{32})$ $V_{B2} = -9.924$

The angle of \mathbf{V}_B is 0 deg if V_B is positive and 180 deg if V_B negative.

6. The velocity of slip is the same as the velocity of pin B .

PROBLEM 6-9a

Statement: The general linkage configuration and terminology for an inverted fourbar slider-crank linkage are shown in Fig P6-3. The link lengths and the values of θ_2 and ω_2 and γ are defined in Table P6-3. For row a , using an analytic method, find the velocities of the pin joints A and B and the velocity of slip at the sliding joint. Draw the linkage to scale and label it before setting up the equations.

Given:

Link lengths:

Link 1 $d := 6$

Link 2 $a := 2$

Link 4 $c := 4$

$\gamma := 90 \cdot \text{deg}$

$\theta_2 := 30 \cdot \text{deg}$

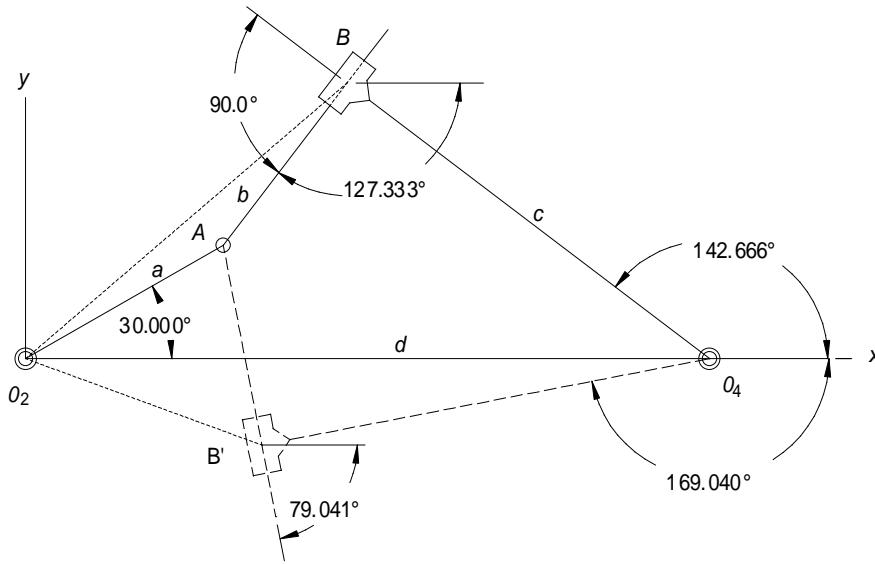
$\omega_2 := 10$

Two argument inverse tangent

$$\text{atan2}(x, y) := \begin{cases} \text{return } 0.5 \cdot \pi \text{ if } x = 0 \wedge y > 0 \\ \text{return } 1.5 \cdot \pi \text{ if } x = 0 \wedge y < 0 \\ \text{return } \text{atan}\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ \text{atan}\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Solution: See Mathcad file P0609a.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$P := a \cdot \sin(\theta_2) \cdot \sin(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \cos(\gamma) \quad P = 1.000$$

$$Q := -a \cdot \sin(\theta_2) \cdot \cos(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \sin(\gamma) \quad Q = -4.268$$

$$R := -c \cdot \sin(\gamma) \quad R = -4.000 \quad T := 2 \cdot P \quad T = 2.000$$

$$S := R - Q \quad S = 0.268 \quad U := Q + R \quad U = -8.268$$

3. Use equation 4.22c to find values of θ_4 for the open and crossed circuits.

$$\text{OPEN} \quad \theta_{41} := 2 \cdot \text{atan2}\left(2 \cdot S, -T + \sqrt{T^2 - 4 \cdot S \cdot U}\right) \quad \theta_{41} = 142.667 \text{ deg}$$

$$\text{CROSSED} \quad \theta_{42} := 2 \cdot \text{atan2}\left(2 \cdot S, -T - \sqrt{T^2 - 4 \cdot S \cdot U}\right) \quad \theta_{42} = -169.041 \text{ deg}$$

4. Use equation 4.18 to find values of θ_3 for the open and crossed circuits.

OPEN $\theta_{31} := \theta_{41} + \gamma$ $\theta_{31} = 232.667 \text{ deg}$

CROSSED $\theta_{32} := \theta_{42} - \gamma$ $\theta_{32} = -259.041 \text{ deg}$

5. Determine the magnitude of the instantaneous "length" of link 3 from equation 4.20a.

OPEN $b_I := \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_{41})}{\sin(\theta_{41} + \gamma)}$ $b_I = 1.793$

CROSSED $b_2 := \left| \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_{42})}{\sin(\theta_{42} + \gamma)} \right|$ $b_2 = 1.793$

6. Determine the angular velocity of link 4 using equation 6.30c:

OPEN $\omega_{41} := \frac{a \cdot \omega_2 \cdot \cos(\theta_2 - \theta_{31})}{b_I + c \cdot \cos(\gamma)}$ $\omega_{41} = -10.292$

CROSSED $\omega_{42} := \frac{a \cdot \omega_2 \cdot \cos(\theta_2 - \theta_{32})}{b_2 + c \cdot \cos(\gamma)}$ $\omega_{42} = 3.639$

7. Determine the velocity of pin A using equation 6.23a:

$$\mathbf{v}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{v}_A = -10.000 + 17.321i \quad |\mathbf{v}_A| = 20.000 \quad \arg(\mathbf{v}_A) = 120.000 \text{ deg}$$

8. Determine the velocity of point B on link 4 using equation 6.31:

OPEN $V_{B4xI} := -c \cdot \omega_{41} \cdot \sin(\theta_{41})$ $V_{B4xI} = 24.966$

$V_{B4yI} := c \cdot \omega_{41} \cdot \cos(\theta_{41})$ $V_{B4yI} = 32.734$

$V_{B4I} := \sqrt{V_{B4xI}^2 + V_{B4yI}^2}$ $V_{B4I} = 41.168$

$\theta_{VB1} := \text{atan2}(V_{B4xI}, V_{B4yI})$ $\theta_{VB1} = 52.667 \text{ deg}$

CROSSED $V_{B4x2} := -c \cdot \omega_{42} \cdot \sin(\theta_{42})$ $V_{B4x2} = 2.767$

$V_{B4y2} := c \cdot \omega_{42} \cdot \cos(\theta_{42})$ $V_{B4y2} = -14.289$

$V_{B42} := \sqrt{V_{B4x2}^2 + V_{B4y2}^2}$ $V_{B42} = 14.555$

$\theta_{VB2} := \text{atan2}(V_{B4x2}, V_{B4y2})$ $\theta_{VB2} = -79.041 \text{ deg}$

9. Determine the slip velocity using equation 6.30a:

OPEN $V_{slip1} := \frac{-a \cdot \omega_2 \cdot \sin(\theta_2) + \omega_{41} \cdot (b_I \sin(\theta_{31}) + c \cdot \sin(\theta_{41}))}{\cos(\theta_{31})}$ $V_{slip1} = 33.461$

CROSSED $V_{slip2} := \frac{-a \cdot \omega_2 \cdot \sin(\theta_2) + \omega_{42} \cdot (b_2 \sin(\theta_{32}) + c \cdot \sin(\theta_{42}))}{\cos(\theta_{32})}$ $V_{slip2} = 33.461$

PROBLEM 6-16c

Statement: The linkage in Figure P6-5a has the dimensions and crank angle given below. Find ω_3 , \mathbf{V}_A , \mathbf{V}_B , and \mathbf{V}_C for the position shown for $\omega_2 = 15 \text{ rad/sec}$ in the direction shown. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)

$$a := 0.80 \text{ in}$$

Distance from A to C

$$R_{ca} := 1.33 \text{ in}$$

Link 3 (A to B)

$$b := 1.93 \text{ in}$$

Angle BAC

$$\delta_3 := 38.6 \text{ deg}$$

Offset

$$c := -0.38 \text{ in}$$

Crank angle:

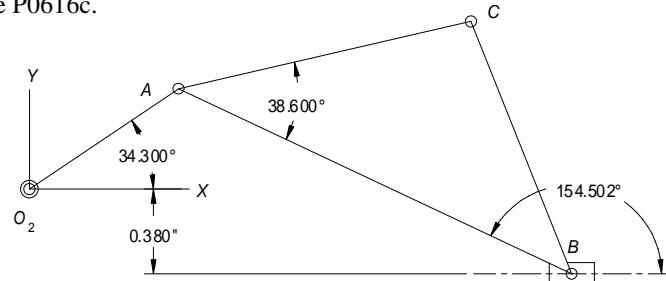
$$\theta_2 := 34.3 \text{ deg}$$

Input crank angular velocity $\omega_2 := 15 \text{ rad/sec}^{-1}$ CCW

Solution: See Figure P6-5a and Mathcad file P0616c.

1. Draw the linkage to scale and label it.

2. Determine θ_3 and d using equation 4.17.



$$\theta_{31} := \arcsin\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi$$

$$\theta_{31} = 154.502 \text{ deg}$$

$$d_I := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_{31}) \quad d_I = 2.403 \text{ in}$$

3. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_{31} := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_{31})} \cdot \omega_2$$

$$\omega_{31} = -5.691 \frac{\text{rad}}{\text{sec}}$$

4. Determine the velocity of pin A using equation 6.23a:

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = (-6.762 + 9.913j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_A| = 12.000 \frac{\text{in}}{\text{sec}} \quad \arg(\mathbf{V}_A) = 124.300 \text{ deg}$$

5. Determine the velocity of pin B using equation 6.22b:

$$\mathbf{V}_B := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{31} \cdot \sin(\theta_{31})$$

$$\mathbf{V}_B = -11.490 \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_B| = 11.490 \frac{\text{in}}{\text{sec}} \quad \arg(\mathbf{V}_B) = 180.000 \text{ deg}$$

6. Determine the velocity of the coupler point C for the open circuit using equations 6.36.

$$\mathbf{V}_{CA} := R_{ca} \cdot \omega_{31} \cdot (-\sin(\pi + \theta_{31} + \delta_3) + j \cdot \cos(\pi + \theta_{31} + \delta_3))$$

$$\mathbf{V}_{CA} = (1.716 - 7.371j) \frac{\text{in}}{\text{sec}}$$

$$\mathbf{V}_C := \mathbf{V}_A + \mathbf{V}_{CA}$$

$$\mathbf{V}_C = (-5.047 + 2.542j) \frac{\text{in}}{\text{sec}} \quad |\mathbf{V}_C| = 5.651 \frac{\text{in}}{\text{sec}} \quad \arg(\mathbf{V}_C) = 153.268 \text{ deg}$$

Note that θ_3 is defined at point B for the slider-crank and at point A for the pin-jointed fourbar. Thus, to use equation 6.36a for the slider-crank, 180 deg must be added to the calculated value of θ_3 .

PROBLEM 6-26

Statement: The linkage in Figure P6-8a has the dimensions and crank angle given below. Find ω_4 , \mathbf{V}_A , and \mathbf{V}_B for the position shown for $\omega_2 = 15 \text{ rad/sec}$ clockwise (CW). Use an analytical method.

Given: Link lengths: Crank angle:

$$\text{Link 2 (}O_2\text{ to }A\text{)} \quad a := 116 \cdot \text{mm} \quad \theta_2 := 62 \cdot \text{deg} \quad \text{Global XY system}$$

$$\text{Link 3 (}A\text{ to }B\text{)} \quad b := 108 \cdot \text{mm} \quad \text{Input crank angular velocity}$$

$$\text{Link 4 (}B\text{ to }O_4\text{)} \quad c := 110 \cdot \text{mm} \quad \omega_2 := -15 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \text{CW}$$

$$\text{Link 1 (}O_2\text{ to }O_4\text{)} \quad d := 174 \cdot \text{mm}$$

$$\begin{aligned} \text{Two argument inverse tangent} \quad \text{atan2}(x, y) := & \begin{cases} \text{return } 0.5 \cdot \pi \text{ if } x = 0 \wedge y > 0 \\ \text{return } 1.5 \cdot \pi \text{ if } x = 0 \wedge y < 0 \\ \text{return } \text{atan}\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ \text{atan}\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases} \end{aligned}$$

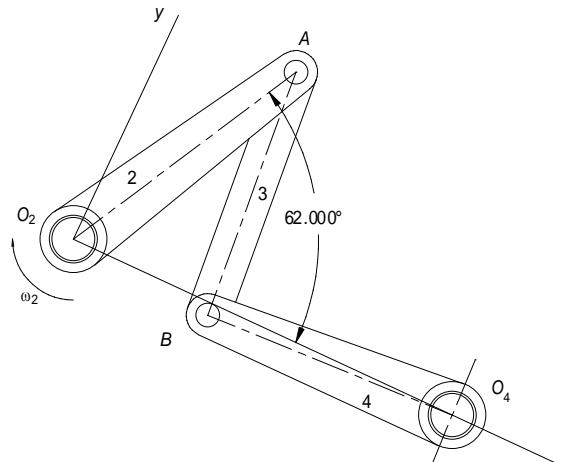
Solution: See Figure P6-8a and Mathcad file P0626.

1. Draw the linkage to scale and label it.
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_1 = 1.5000$$

$$K_2 := \frac{d}{c} \quad K_2 = 1.5818$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.7307$$



$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.0424 \quad B = -1.7659 \quad C = 2.0186$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{42} := 2 \cdot \left(\text{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right) \right) \quad \theta_{42} = 182.681 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.6111 \quad K_5 = -1.7280$$

$$D := \cos(\theta_2) - K_1 + K_4 \cos(\theta_2) + K_5 \quad D = -2.0021$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.7659$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 0.0589$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_{32} = 2 \cdot \left(\text{atan}2\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F}\right) \right) \quad \theta_{32} = 275.133 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_{32} = \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{42} - \theta_2)}{\sin(\theta_{32} - \theta_{42})} \quad \omega_{32} = -13.869 \frac{\text{rad}}{\text{sec}}$$

$$\omega_{42} = \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{32})}{\sin(\theta_{42} - \theta_{32})} \quad \omega_{42} = 8.654 \frac{\text{rad}}{\text{sec}}$$

7. Determine the velocity of points A and B for the crossed circuit using equations 6.19.

$$V_A = a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_A = (1536.329 - 816.881j) \frac{\text{mm}}{\text{sec}} \quad |V_A| = 1740.000 \frac{\text{mm}}{\text{sec}} \quad \arg(V_A) = -28.000 \text{ deg}$$

$$V_B = c \cdot \omega_{42} \cdot (-\sin(\theta_{42}) + j \cdot \cos(\theta_{42}))$$

$$V_B = (44.524 - 950.875j) \frac{\text{mm}}{\text{sec}} \quad |V_B| = 951.917 \frac{\text{mm}}{\text{sec}} \quad \arg(V_B) = -87.319 \text{ deg}$$

PROBLEM 6-30

Statement: The linkage in Figure P6-8b has the dimensions and crank angle given below. Find ω_4 , \mathbf{V}_A , and \mathbf{V}_B for the position shown for $\omega_2 = 20 \text{ rad/sec}$ counterclockwise (CCW). Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)

$$a := 40 \text{ mm}$$

$\theta_{21} = 57^\circ \text{ deg}$ Global XY system

Link 3 (A to B)

$$b := 96 \text{ mm}$$

Input crank angular velocity

Link 4 (B to O_4)

$$c := 122 \text{ mm}$$

$$\omega_2 := 20 \text{ rad/sec}^{-1}$$

Link 1 (O_2 to O_4)

$$d := 162 \text{ mm}$$

Coordinate rotation angle

$$\alpha := -36^\circ \text{ deg}$$

Global XY system to local xy system

Two argument inverse tangent

$$\text{atan2}(x, y) :=$$

$$\begin{cases} \text{return } 0.5\pi \text{ if } x = 0 \wedge y > 0 \\ \text{return } 1.5\pi \text{ if } x = 0 \wedge y < 0 \\ \text{return } \text{atan}\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ \text{return } \text{atan}\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Solution: See Figure P6-8b and Mathcad file P0630.

1. Draw the linkage to scale and label it.
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_2 := \frac{d}{c}$$

$$K_1 = 4.0500$$

$$K_2 = 1.3279$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_3 = 3.4336$$

$$\theta_2 := \theta_{21} - \alpha$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.5992 \quad B = -1.9973 \quad C = 7.6054$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(\text{atan2}(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}) \right) - 2 \cdot \pi \quad \theta_4 = 132.386 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b}$$

$$K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 1.6875$$

$$K_5 = -2.8875$$

$$D := \cos(\theta_2) - K_1 + K_4 \cos(\theta_2) + K_5$$

$$D = -7.0782$$

$$E := -2 \cdot \sin(\theta_2)$$

$$E = -1.9973$$

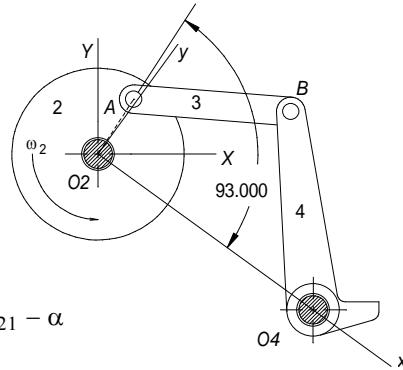
$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

$$F = 1.1265$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(\text{atan2}(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}) \right) - 2 \cdot \pi \quad \theta_3 = 31.504 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.



$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \quad \omega_3 = -5.385 \frac{\text{rad}}{\text{sec}}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad \omega_4 = 5.868 \frac{\text{rad}}{\text{sec}}$$

7. Determine the velocity of points A and B for the open circuit using equations 6.19.

$$\mathbf{V_A} := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V_A} = (-798.904 - 41.869j) \frac{\text{mm}}{\text{sec}} \quad |\mathbf{V_A}| = 800.000 \frac{\text{mm}}{\text{sec}} \quad \arg(\mathbf{V_A}) = -177.000 \text{deg}$$

$$\mathbf{V_B} := c \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4))$$

$$\mathbf{V_B} = (-528.774 - 482.608j) \frac{\text{mm}}{\text{sec}} \quad |\mathbf{V_B}| = 715.900 \frac{\text{mm}}{\text{sec}} \quad \arg(\mathbf{V_B}) = -137.614 \text{deg}$$

PROBLEM 6-34

Statement: The offset slider-crank linkage in Figure P6-8f has the dimensions and crank angle given below. Find \mathbf{V}_A and \mathbf{V}_B for the position shown for $\omega_2 = 25 \text{ rad/sec CW}$. Use an analytical method.

Given: Link lengths:

$$\text{Link 2} \quad a := 63 \cdot \text{mm} \quad \text{Crank angle:} \quad \theta_2 := 141 \cdot \text{deg}$$

$$\text{Link 3} \quad b := 130 \cdot \text{mm} \quad \text{Local } xy \text{ coordinate system}$$

$$\text{Offset} \quad c := -52 \cdot \text{mm} \quad \text{Input crank angular velocity}$$

$$\omega_2 := -25 \cdot \text{rad.sec}^{-1}$$

Solution: See Figure P6-8f and Mathcad file P0634.

1. Draw the linkage to a convenient scale.
2. Determine θ_3 and d using equations 4.16 for the crossed circuit.

$$\theta_3 := \arcsin\left(\frac{a \cdot \sin(\theta_2) - c}{b}\right) \quad \theta_3 = 44.828 \text{ deg}$$

$$d := a \cdot \cos(\theta_2) - b \cdot \cos(\theta_3) \quad d = -141.160 \text{ mm}$$

3. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3)} \cdot \omega_2 \quad \omega_3 = 13.276 \frac{\text{rad}}{\text{sec}}$$

4. Determine the velocity of pin A using equation 6.23a:

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = (991.180 + 1224.005i) \frac{\text{mm}}{\text{sec}} \quad |\mathbf{V}_A| = 1575.000 \frac{\text{mm}}{\text{sec}} \quad \arg(\mathbf{V}_A) = 51.000 \text{ deg}$$

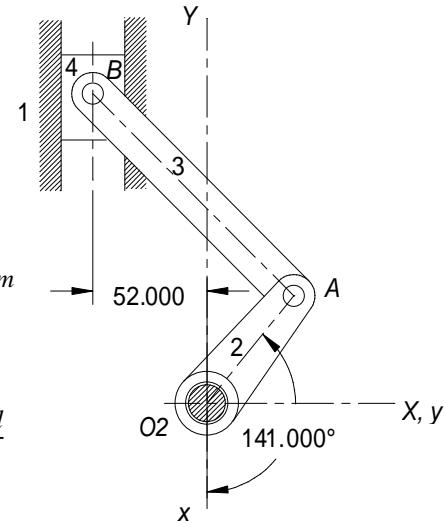
$$\text{In the global coordinate system,} \quad \theta_{VA} := \arg(\mathbf{V}_A) - 90 \cdot \text{deg} \quad \theta_{VA} = -39.000 \text{ deg}$$

5. Determine the velocity of pin B using equation 6.22b:

$$V_B := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_3 \cdot \sin(\theta_3) \quad V_B = 2207.849 \frac{\text{mm}}{\text{sec}}$$

$$\mathbf{V}_B := V_B$$

$$\text{In the global coordinate system,} \quad \theta_{VB} := \arg(\mathbf{V}_B) - 90 \cdot \text{deg} \quad \theta_{VB} = -90.000 \text{ deg}$$



PROBLEM 6-41

Statement: The linkage in Figure P6-8g has the dimensions and crank angle given below. Find ω_4 , \mathbf{V}_A , and \mathbf{V}_B for the position shown for $\omega_2 = 15 \text{ rad/sec}$ clockwise (CW). Use an analytical method.

Given: Link lengths:

$$\begin{array}{lll} \text{Link 2 } (O_2 \text{ to } A) & a := 49 \cdot \text{mm} & \text{Link 2 } (O_2 \text{ to } C) \\ \text{Link 3 } (A \text{ to } B) & b := 100 \cdot \text{mm} & \text{Link 5 } (C \text{ to } D) \\ \text{Link 4 } (B \text{ to } O_4) & c := 153 \cdot \text{mm} & \text{Link 6 } (D \text{ to } O_6) \\ \text{Link 1 } (O_2 \text{ to } O_4) & d := 87 \cdot \text{mm} & \text{Link 1 } (O_2 \text{ to } O_6) \end{array}$$

$$\text{Crank angle: } \theta_2 := 148 \cdot \text{deg} \quad \text{Local } xy \text{ system}$$

$$\text{Input crank angular velocity } \omega_2 := -15 \cdot \text{rad.sec}^{-1}$$

$$\text{Two argument inverse tangent } \text{atan2}(x, y) := \begin{cases} \text{return } 0.5 \cdot \pi \text{ if } x = 0 \wedge y > 0 \\ \text{return } 1.5 \cdot \pi \text{ if } x = 0 \wedge y < 0 \\ \text{return } \text{atan}\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ \text{atan}\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Solution: See Figure P6-8g and Mathcad file P0641.

1. Draw the linkage to scale and label it.
2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c}$$

$$K_1 = 1.7755 \quad K_2 = 0.5686$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.5592$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.5821 \quad B = -1.0598 \quad C = 4.6650$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{42} := 2 \cdot \left(\text{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right) \right) \quad \theta_{42} = 208.876 \text{ deg}$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

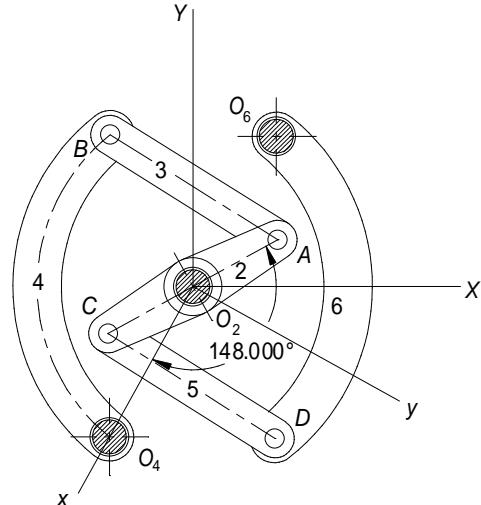
$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 0.8700 \quad K_5 = 0.3509$$

$$D := \cos(\theta_2) - K_1 + K_4 \cos(\theta_2) + K_5 \quad D = -3.0104$$

$$E := -2 \cdot \sin(\theta_2) \quad E = -1.0598$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = 2.2367$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.



$$\theta_{32} := 2 \cdot \left(\text{atan2}\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_{32} = 266.892 \text{ deg}$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_{32} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{42} - \theta_2)}{\sin(\theta_{32} - \theta_{42})} \quad \omega_{32} = -7.570 \frac{\text{rad}}{\text{sec}}$$

$$\omega_{42} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{32})}{\sin(\theta_{42} - \theta_{32})} \quad \omega_{42} = -4.959 \frac{\text{rad}}{\text{sec}}$$

7. Determine the velocity of points B and A for the crossed circuit using equations 6.19.

$$\mathbf{V}_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V}_A = (389.491 + 623.315j) \frac{\text{mm}}{\text{sec}} \quad |\mathbf{V}_A| = 735.000 \frac{\text{mm}}{\text{sec}} \quad \arg(\mathbf{V}_A) = 58.000 \text{ deg}$$

$$\mathbf{V}_B := c \cdot \omega_{42} \cdot (-\sin(\theta_{42}) + j \cdot \cos(\theta_{42}))$$

$$\mathbf{V}_B = (-366.389 + 664.362j) \frac{\text{mm}}{\text{sec}} \quad |\mathbf{V}_B| = 758.694 \frac{\text{mm}}{\text{sec}} \quad \arg(\mathbf{V}_B) = 118.876 \text{ deg}$$

PROBLEM 6-49

Statement: Figure P6-12 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of the coupler point P at 2-deg increments of crank angle over the maximum range of motion possible. Check your results with program FOURBAR.

Units: $rpm := 2\pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

$$\text{Link 2 } (O_2 \text{ to } A) \quad a := 0.785 \text{ in}$$

$$\text{Link 4 } (B \text{ to } O_4) \quad c := 0.950 \text{ in}$$

$$\text{Coupler point:} \quad R_{pa} := 1.09 \text{ in}$$

$$\text{Crank speed:} \quad \omega_2 := 20 \text{ rpm}$$

$$\text{Two argument inverse tangent} \quad atan2(x, y) := \begin{cases} \text{return } 0.5\pi \text{ if } x = 0 \wedge y > 0 \\ \text{return } 1.5\pi \text{ if } x = 0 \wedge y < 0 \\ \text{return } \text{atan}\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ \text{atan}\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Solution: See Figure P6-12 and Mathcad file P0649.

1. Draw the linkage to scale and label it.
2. Using the geometry defined in Figure 3-1a in the text, determine the input crank angles (relative to the line O_2O_4) at which links 2 and 3, and 3 and 4 are in toggle.

$$\theta_{20} := \text{acos}\left[\frac{a^2 + d^2 - (b + c)^2}{2 \cdot a \cdot d}\right]$$

$$\theta_{20} = 158.286 \text{ deg}$$

$$\theta_2 := -\theta_{20}, -\theta_{20} + 2 \cdot \text{deg..} \theta_{20}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a}$$

$$K_1 = 0.6930$$

$$K_2 := \frac{d}{c}$$

$$K_2 = 0.5726$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_3 = 1.1317$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2}\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)}\right) \right)$$

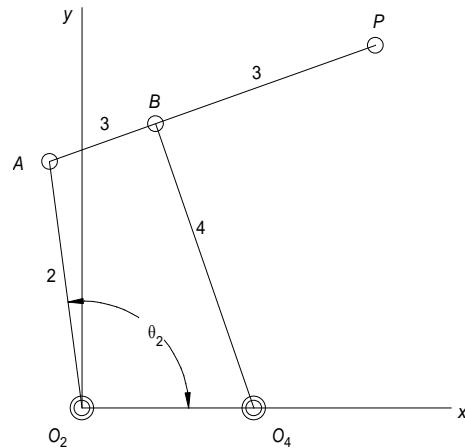
5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b}$$

$$K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 1.5281$$

$$K_5 = -0.2440$$



$$D(\theta_2) := \cos(\theta_2) - K_I + K_4 \cdot \cos(\theta_2) + K_5 \quad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_I + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan}2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of point A using equations 6.19.

$$\mathbf{V}_A(\theta_2) := a \cdot \omega_2 (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

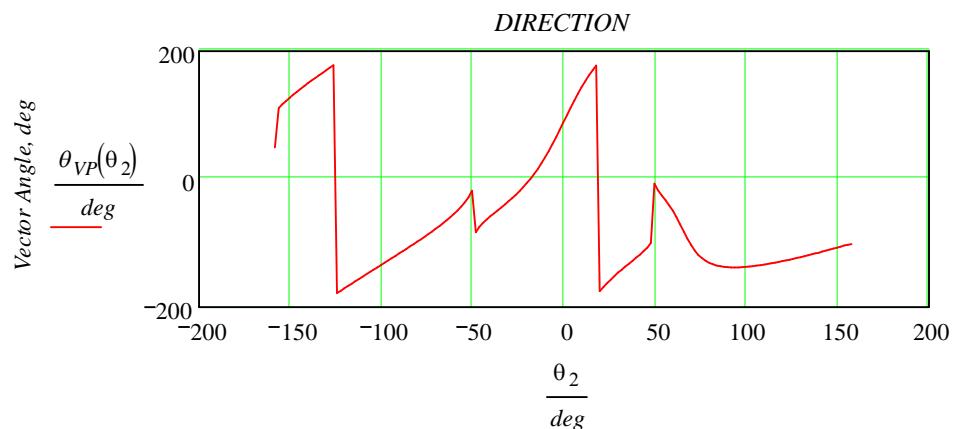
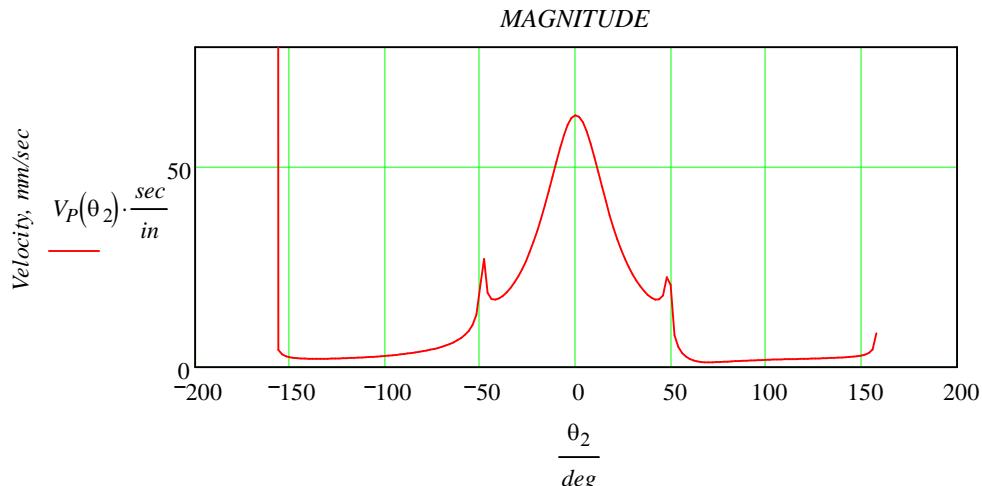
9. Determine the velocity of the coupler point P using equations 6.36.

$$\mathbf{V}_{PA}(\theta_2) := R_{pa} \cdot \omega_{31}(\theta_2) \cdot (-\sin(\theta_{31}(\theta_2) + \delta_3) + j \cdot \cos(\theta_{31}(\theta_2) + \delta_3))$$

$$\mathbf{V}_P(\theta_2) := \mathbf{V}_A(\theta_2) + \mathbf{V}_{PA}(\theta_2)$$

10. Plot the magnitude and direction of the coupler point P.

Magnitude: $V_P(\theta_2) := |\mathbf{V}_P(\theta_2)|$ Direction: $\theta_{VP}(\theta_2) := \arg(\mathbf{V}_P(\theta_2))$



PROBLEM 6-55a

Statement: Figure P6-18 shows a powder compaction mechanism. Calculate its mechanical advantage for the position shown.

Given: Link lengths:

$$\text{Link 2 (A to B)} \quad a := 105 \cdot \text{mm} \quad \text{Offset} \quad c := 27 \cdot \text{mm}$$

$$\text{Link 3 (B to D)} \quad b := 172 \cdot \text{mm}$$

Distance to force application:

$$\text{Link 2 (AC)} \quad r_{in} := 301 \cdot \text{mm}$$

$$\text{Position of link 2: } \theta_2 := 44 \cdot \text{deg} \quad \text{Let } \omega_2 := 1 \cdot \text{rad} \cdot \text{sec}^{-1}$$

Solution: See Figure P6-18 and Mathcad file P0655a.

1. Draw the linkage to scale and label it.

2. Determine θ_3 using equation 4.17.

$$\theta_3 := \arcsin\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi$$

$$\theta_3 = 164.509 \cdot \text{deg}$$

3. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3)} \cdot \omega_2$$

4. Determine the velocity of pin D using equation 6.22b:

$$V_D := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_3 \cdot \sin(\theta_3) \quad \text{Positive upward}$$

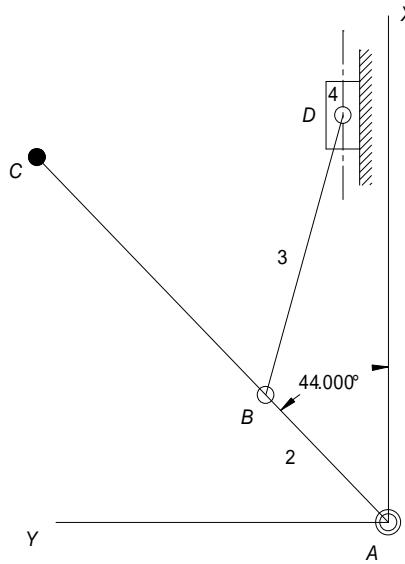
5. Calculate the velocity of point C using equation 6.23a:

$$\mathbf{v}_C := r_{in} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_C := |\mathbf{v}_C|$$

6. Calculate the mechanical advantage using equation 6.13.

$$m_A := \frac{V_C}{|V_D|} \quad m_A = 3.206$$



PROBLEM 6-59a

Statement: Figure P6-22 shows a fourbar toggle clamp used to hold a workpiece in place by clamping it at D. The linkage will toggle when link 2 reaches 90 deg. For the dimensions given below, calculate its mechanical advantage for the position shown.

Given:

Link lengths:

$$\text{Link } 2 (O_2A) \quad a := 70\text{-mm} \quad \text{Link } 3 (AB) \quad b := 35\text{-mm}$$

$$\text{Link } 4 (O_4B) \quad c := 34\text{-mm} \quad \text{Link } 1 (O_2O_4) \quad d := 48\text{-mm}$$

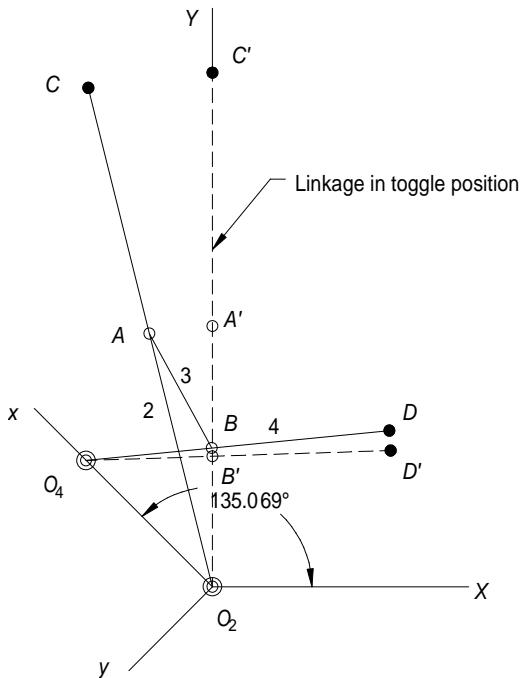
Distance to force application:

$$\text{Link } 2 (O_2C) \quad r_{in} := 138\text{-mm} \quad \text{Link } 4 (O_4D) \quad r_{out} := 82\text{-mm}$$

$$\text{Initial position of link 2: } \theta_{20} := 104\text{-deg} \quad \text{Global XY system}$$

Solution: See Figure P6-22 and Mathcad file P0659a.

1. Draw the mechanism to scale and label it. To establish the position of O_4 with respect to O_2 (in the global coordinate frame), draw the linkage in the toggle position with $\theta_2 = 90$ deg. The fixed pivot O_4 is then 48 mm from O_2 and 34 mm from B' (see layout).



2. Calculate the value of θ_2 in the local coordinate system (required to calculate θ_3 and θ_4).

$$\text{Rotation angle of local } xy \text{ system to global } XY \text{ system: } \alpha := 135.069\text{-deg}$$

$$\theta_2 := \theta_{20} - \alpha \quad \theta_2 = -31.069 \text{ deg}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad K_1 = 0.6857 \quad K_2 := \frac{d}{c} \quad K_2 = 1.4118$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c} \quad K_3 = 1.4989$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = 0.4605 \quad B = 1.0321 \quad C = 0.1189$$

4. Use equation 4.10b to find value of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(\operatorname{atan}2 \left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \quad \theta_4 = -129.480 \text{ deg}$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \quad K_4 = 1.3714$$

$$K_5 = -1.4843$$

$$D := \cos(\theta_2) - K_1 + K_4 \cos(\theta_2) + K_5 \quad D = -0.1388$$

$$E := -2 \cdot \sin(\theta_2) \quad E = 1.0321$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \quad F = -0.4804$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(\operatorname{atan}2 \left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \quad \theta_3 = -196.400 \text{ deg}$$

7. Referring to Figure 6-10, calculate the values of the angles v and μ .

$$v := \theta_2 - \theta_3 \quad v = 165.331 \text{ deg}$$

If $v > 90$ deg, subtract it from 180 deg.

$$v := \operatorname{if}(v > 90 \cdot \text{deg}, 180 \cdot \text{deg} - v, v) \quad v = 14.669 \text{ deg}$$

$$\mu := \theta_4 - \theta_3 \quad \mu = 66.920 \text{ deg}$$

If $\mu > 90$ deg, subtract it from 180 deg.

$$\mu := \operatorname{if}(\mu > 90 \cdot \text{deg}, 180 \cdot \text{deg} - \mu, \mu) \quad \mu = 66.920 \text{ deg}$$

8. Using equation 6.13e, calculate the mechanical advantage of the linkage in the position shown.

$$m_A := \frac{c \cdot \sin(\mu)}{a \cdot \sin(v)} \cdot \frac{r_{in}}{r_{out}} \quad m_A = 2.970$$